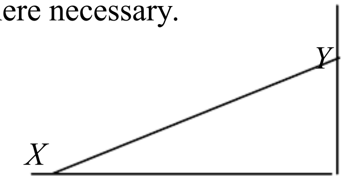
MECHANICS (C) UNIT 2 TEST PAPER 4

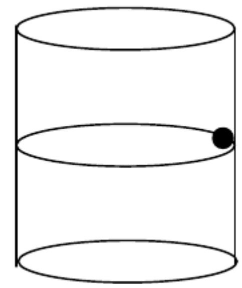
Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A stick of mass 0·75 kg is at rest with one end X on a rough horizontal floor and the other end Y leaning against a smooth vertical wall. The coefficient of friction between the stick and the floor is 0·6. Modelling the stick as a uniform rod, find the smallest angle that the stick can make with the floor before it starts to slip. [6]



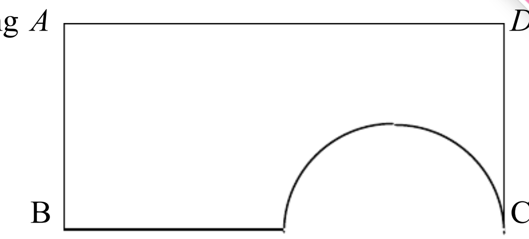
2. An engine of mass 20 000 kg climbs a hill inclined at 10^0 to the horizontal. The total non-gravitational resistance to its motion has magnitude 35 000 N and the maximum speed of the engine on the hill is 15 ms^{-1}.
   (i) Find, in kW, the maximum rate at which the engine can work. [4]
   (ii) Find the maximum speed of the engine when it is travelling on a horizontal track against the same non-gravitational resistance as before. [3]

3. A motorcyclist rides in a cylindrical well of radius 5 m. He maintains a horizontal circular path at a constant speed of 10 ms^{-1}. The coefficient of friction between the wall and the wheels of the cycle is μ.
   Modelling the cyclist and his machine as a particle in contact with the wall, show that he will not slip downwards provided that $\mu \geq 0{\cdot}49$. [7]



4. A ball of mass 0·15 kg is dropped from rest at a height of 40 cm above horizontal ground. It falls vertically, hits the ground and rebounds vertically upwards, coming to instantaneous rest 0·2 seconds later. Calculate
   (i) the coefficient of restitution between the ball and the ground, [6]
   (ii) the impulse which the ground exerts on the ball when it rebounds. [2]

5. A rectangular piece of cardboard *ABCD*, measuring 30 cm by 12 cm, has a semicircle of radius 5 cm removed from it as shown.



Given that the centre of mass of a uniform semi-circular lamina of radius a is at a distance from the bounding diameter,

(i) calculate the distances of the centre of mass of the remaining piece of cardboard from *AB* and from *BC*. [6]

The remaining cardboard is suspended from *A* and hangs in equilibrium.

(ii) Find the angle made by *AB* with the vertical. [3]

6. Three particles *A, B* and *C*, of equal size and each of mass m, are at rest on the same straight line on a smooth horizontal surface. The coefficient of restitution between *A* and *B*, and between *B* and *C*, is e.

A is projected with speed 7 ms^{-1} and strikes *B* directly. *B* then collides with *C*, which starts to move with speed 4 ms^{-1}.

Calculate the value of e. [10]

7. A golf ball is hit with initial velocity u ms^{-1} at an angle of 45^0 above the horizontal. The ball passes over a building which is 15 m tall, at a distance of 30 m horizontally from the point where the ball was hit.

(i) Find the smallest possible value of u. [6]

When u has this minimum value,

(ii) show that the ball does not rise higher than the top of the building. [3]

(iii) Deduce the total horizontal distance travelled by the ball before it hits the ground. [2]

(iv) Briefly describe two modelling assumptions that you have made. [2]

<u>MECHANICS 2 (C) TEST PAPER 4 : ANSWERS AND MARK SCHEME</u>

1.	Reaction at $X = R = 0{\cdot}75g$ Friction $= 0{\cdot}6R = 0{\cdot}45g$	B1 B1	
	Reaction at $Y = S = 0{\cdot}45g$	B1	
	M(X) : $0{\cdot}75g(a \cos \alpha) = 0{\cdot}45g(2a \sin \alpha)$ $\tan \alpha = 0.83$ $\alpha = 39{\cdot}8^0$	M1 A1 A1	6
2.	(i) $P = 15(35\,000 + 20\,000g \sin 10^0) = 1\,035\,525{\cdot}6$ W$\approx$1040 kW	M1 M1 A1 A1	
	(ii) $1\,035\,525{\cdot}6 = v \times 35\,000$ $v = 29{\cdot}6$ ms^{-2}	M1 A1 A1	7

3. Frictional force $F = mg$; normal reaction $R = m(10^2/5) = 20m$ — M1 A1 A1

$F/R = g/20 = 0{\cdot}49$ No slip if $F \le \mu R$ $\mu \ge 0{\cdot}49$ — M1 A1 M1 A17

4. (i) $v^2 = 2gs$, so hits ground with speed $\sqrt{(0{\cdot}8g)} = 2{\cdot}8\ \text{ms}^{-1}$ ie u — M1 A1

$0 = u - gt$, so rebound speed $u = 0{\cdot}2g = 1{\cdot}96\ \text{ms}^{-1}$ ie v — M1 A1

$e = 1{\cdot}96 \div 2{\cdot}8 = 0{\cdot}7$ — M1 A1

(ii) Impulse $= m(v - u) = 0{\cdot}15(4{\cdot}76) = 0{\cdot}714$ Ns — M1 A1 8

5. (i) $360(15) = 12{\cdot}5\pi(25) + (360 - 12{\cdot}5\pi)\bar{x}$ $\bar{x} = 13{\cdot}8$ — M1 A1 A1

$360(6) = 12{\cdot}5\pi(20/3\pi) + (360 - 12{\cdot}5\pi)\bar{y}$ $\bar{y} = 6{\cdot}47$ — M1 A1 A1

(ii) $\tan\alpha = 13{\cdot}78 \div (12 - 6{\cdot}475) = 2{\cdot}494$ $\alpha = 68{\cdot}2^0$ — M1 A1 A1 9

6. $v_A + v_B = 7$ $(v_B - v_A)/(0 - 7) = -e$ $2v_B = 7(e + 1)$ — B1 M1 A1

$4 + v'_B = v_B$ $(4 - v'_B)/(0 - vB) = -e$ $8 = v_B(e + 1)$ — B1 M1 A1 A1

$16 = 7(e + 1)2$ $e = 0{\cdot}512$ — M1 A1 A1 10

7. (i) $x = (u\cos 45^0)t$, $y = (u\sin 45^0)t - 4{\cdot}9t^2$ $y = x - \dfrac{g}{u^2}x^2$ — M1 M1 A1

Need $15 \le 30 - 900\dfrac{g}{u^2}$ $u^2 \ge 60g$ $u \ge 24{\cdot}2\ \text{ms}^{-1}$ — M1 A1 A1

(ii) At max. height, $u\sin 45^0 - gt = 0$ $t = 1{\cdot}75$ $y_{max} = 15$ — M1 A1 A1

(iii) When $t = 3{\cdot}5$, $x = 60$ m — M1 A1

(iv) Ball modelled as particle; constant gravity; etc. — B1 B1 13